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MAT3420 Solution of Mandatory assignment 1

Problem 1:

Part (a):

For the state $|0\rangle|0\rangle$, the von Neumann measurement rule is as follows:

- Probability of outcome $|\theta\rangle|\theta\rangle$: $|\langle\theta|0\rangle|2=\cos2(\theta)|\langle\theta|0\rangle|2=\cos2(\theta)$
- Probability of outcome $|\theta+\pi2\rangle|\theta+2\pi\rangle$: $|\langle\theta+\pi2|0\rangle|2=\sin2(\theta)|\langle\theta+2\pi|0\rangle|2=\sin2(\theta)$

For the state $|1\rangle|1\rangle$, the probabilities are interchanged.

Part (b):

- $\langle00|P\theta|00\rangle=\cos2(\theta)\langle00|P\theta|00\rangle=\cos2(\theta)$
- $\langle11|P\theta|11\rangle=\sin2(\theta)\langle11|P\theta|11\rangle=\sin2(\theta)$
- $\langle00|P\theta|11\rangle=0\langle00|P\theta|11\rangle=0$
- $\langle11|P\theta|00\rangle=0\langle11|P\theta|00\rangle=0$

Part (c):

Given the decomposition $I=P\theta+P\theta+\pi2I=P\theta+P\theta+2\pi$:

- Probability of outcome "θ": 1221
- Probability of outcome "$\theta + \frac{\pi}{2}$": 1221

Part (d):

The probabilities for outcomes "ϕ" and "$\phi + \pi/2$" are:

- Probability of outcome "ϕ": $\cos2(\theta-\phi)\cos2(\theta-\phi)$
- Probability of outcome "$\phi + \pi/2$": $\sin2(\theta-\phi)\sin2(\theta-\phi)$

Problem 2:

Part (a):

The two circuits for CZ gate when $U=ZU=Z$ are equivalent because:

- Circuit 1: ZZ
- Circuit 2: H HHH

Part (b):

For $U=XU=X$, the CCU gate is implemented as described. Similarly, for CCZ:

- Circuit for CCZ: CCNOT12 CCNOT23 H2 H3CCNOT12CCNOT23H2H3